INTERNAL GRAVITY WAVES IN NON-STATIONARY STRATIFIED MEDIUM

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ABSTRACT.
The paper is devoted to the research of the processes of disturbance and propagation of the internal gravity waves within the vertically stratified non-stationary medium, to development of the asymptotic methods being by the generalization of the space-time ray-tracing method (the method of the geometrical optics, the modified WKBJ method). Numerical results obtained with the use of asymptotic formulas for the real parameters of the ocean are presented.

PROBLEM FORMULATION AND BASIC EQUATIONS.

Under the real oceanic conditions the Vaisala-Brunt frequency $N^2(z,t)=-g\partial\ln\rho/\partial z$, where g is the free-fall acceleration, ρ is the non-perturbated ocean density, which defines the basic characteristics of internal gravity waves, shall not depend solely on space variables *(x,y,z)*, but also on the time *t* [1-3]. The most characteristic types of $N^2(z,t)$ time-to-time variability are the thermocline going up or down and changing its width, etc. There is a number of time scales for variations of hydro-physical fields in the oceans and seas: a small-scale with periods of about 10 minutes, a meso-scale with periods of about a day (twenty-four hours), as well as synoptical and global variations with periods of a few months to a few years. In what follows we shall analyze the internal gravity field propagation in non-stationary mediums with parameter variation periods of a day and over, which allows us to use the geometric optics approximation because the period of internal gravity waves is tens of minutes and less. The system of linearized equations of hydrodynamics, when the non-perturbated density ρ depends on variables z and t, reduces to a single equation, for example, for the vertical velocity equation: $\left(\frac{\partial}{\partial t}+\frac{\partial\ln\rho}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\Delta+\frac{\partial^2}{\partial z^2}\right)+\frac{\partial\ln\rho}{\partial z}\frac{\partial^2}{\partial t\partial z}\right]W=g\frac{\partial\ln\rho}{\partial z}\Delta W$, $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$. If $\partial\ln\rho/\partial z$ is neglected, we obtain an equation in the Boussinesq approximation: $\left(\frac{\partial}{\partial t}+\frac{\partial\ln\rho}{\partial t}\right)\left[\frac{\partial}{\partial t}\left(\Delta+\frac{\partial^2}{\partial z^2}\right)\right]W+N^2(z,t)\Delta W=0$. It appears natural to neglect as well the member with $\partial\ln\rho/\partial t$, which would correspond to a consequent application of the Boussinesq hypothesis. It means the density characterizing the liquid's inert mass can be assumed constant. Then we have: $\frac{\partial^2}{\partial t^2}(\Delta+\frac{\partial^2}{\partial z^2})W+N^2(z,t)\Delta W=0$. The resulting equation differs from a standard equation of internal gravity waves in a stationary stratified medium just by the time t parametrical inclusion into the Vaisala-Brunt frequency.

ASYMPTOTIC FORMS OF SOLUTION.

The asymptotic solution is found in the form of a sum of modes with every one of them propagating independently of each other (the adiabatic approximation). We are going to examine a single individually taken mode while omitting its index. Next we focus solely on the space region near the wave front which means that we consider the time t as being close to the arrival time of the wave front, henceforth denoted by τ, i.e., we use a weakly dispersive approximation. Consider the wave propagation in a layer of stratified medium $-H<z<0$ with the Vaisala-Brunt frequency $N^2(z,t)$. We shall seek the solution with boundary conditions $W = 0, \quad z = 0, -H$ in the form [2,3]: $W = W_0 + W_1 + O(\varepsilon^{2p})$, $W_0 = (A(\varepsilon x, \varepsilon y, \tau, z) + \dfrac{\partial A(\varepsilon x, \varepsilon y, \tau, z)}{\partial \tau}(\varepsilon t - \tau) + \cdots)F_0(\varphi)$, $F_{m+1}{}'(\varphi) = F_m(\varphi), \quad \tau = \tau(\varepsilon x, \varepsilon y)$, $W_1 = (B(\varepsilon x, \varepsilon y, \tau, z) + \dfrac{\partial B(\varepsilon x, \varepsilon y, \tau, z)}{\partial \tau}(\varepsilon t - \tau) + \cdots)F_1(\varphi)$, where $p = 2/3, \quad F_0(\varphi) = Ai'(\varphi)$ is the Airy derivative having its argument $\varphi = \alpha(\varepsilon x, \varepsilon y)(\varepsilon t - \tau(\varepsilon x, \varepsilon y))\varepsilon^{-p}$ of the order of unit. The function τ defines the wave front position, function α describes the evolution of the Airy wave width, the small parameter ε specifies "slow variables". Since our focus is only on "slow times" εt being close to the time of the wave front arrival τ, then all functions preceding functions F_m, are given in the form of Taylor series by $\varepsilon t - \tau \approx \varepsilon^p$ powers. Let $N^2(z, \varepsilon t)$ be written as: $N^2(z, \varepsilon t) = N^2(z, \tau) + \dfrac{\partial N^2(z, \tau)}{\partial \tau}(\varepsilon t - \tau) + O(\varepsilon^{2p})$. We consider the far internal gravity waves generated by moving source. After complicated analytical calculations, we can obtain an expression for the term W_0 in the form: $W_0 = \dfrac{c^{5/2}(\tau_0)\,\alpha^2(x,y)\,f(z,\tau)}{2c^{3/2}(\tau)\,R^{1/2}(x,y,t)(V^2 - c^2(\tau_0))^{1/4}}\dfrac{\partial f(z_0, \tau_0)}{\partial z_0} Ai'\left(\alpha(x,y)\dfrac{t - \tau(x,y)}{\varepsilon^{2/3}}\right)$, where c, f - eigenvalue and eigenfunction of internal gravity vertical spectral problem, *V*- source speed, z_0 - depth of source motion, *R(x,y,t)* - some function that are determined by the parameters of the problem [1-3].

NUMERICAL RESULTS AND DISCUSSION.

The figures demonstrate the numerical results of internal gravity wave calculations for typical oceanic parameters. The fig.1 shows a system of rays (thin line), caustics (bold line) generated by source moving in a non-stationary stratified ocean. The fig.2-4 demonstrates a evolution of internal gravity wave packet in a non-stationary stratified ocean. within the system of coordinates that is in motion together with the disturbing source. Time interval for calculations is equal 2 hours. At that it's evident that if there were no Vaisala-Brunt time-to-time frequency variations such a wave coordinate would be stationary. Numerical results show that internal gravity waves dynamic in the ocean is substantially influenced by non-stationarity of hydro-physical fields. The obtained asymptotic solutions are uniform and allow far internal gravity wave fields to be described both near and far from turning points.

The universal character of the asymptotic method proposed for modeling far internal gravity fields makes it possible to effectively calculate wave fields and, in addition, qualitatively analyze the obtained solutions. This method offers broad opportunities for the analysis of wave fields on a large scale, which is important for developing correct mathematical models of wave dynamics and for assessing in situ measurements of wave fields in the ocean. The particular role of the proposed asymptotic methods is determined by the fact that the parameters of natural stratified media are usually known approximately and attempts at their adequate numerical solution using the initial equations of hydrodynamics and such parameters may result in a notable loss of accuracy for the results obtained. In addition to their fundamental significance, the obtained asymptotic models are also important for applied investigations, since the proposed method of geometrical optics allows solution of a wide spectrum of problems related to modeling wave fields. In such a situation, the description and analysis of wave dynamics may be realized through developing asymptotic models and using analytical methods for their solution based on the proposed WKBJ modified method.

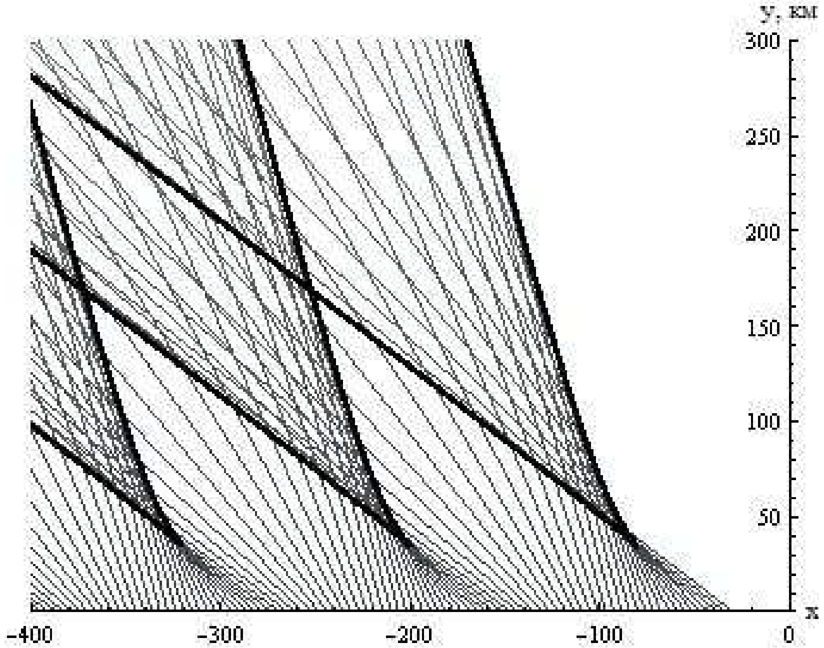

Fig. 1. Rays and caustics in stratified non-stationary medium.

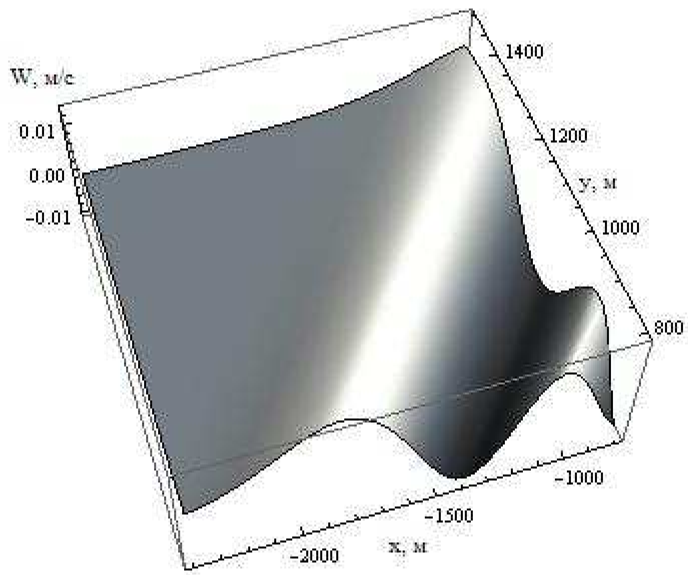

Fig. 2. Evolution of internal gravity wave packet in non-stationary stratified ocean.

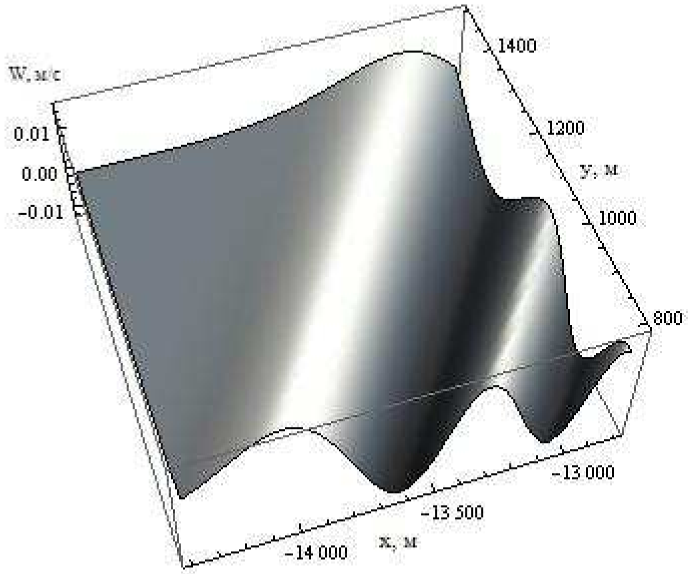

Fig. 3. Evolution of internal gravity wave packet in non-stationary stratified ocean.

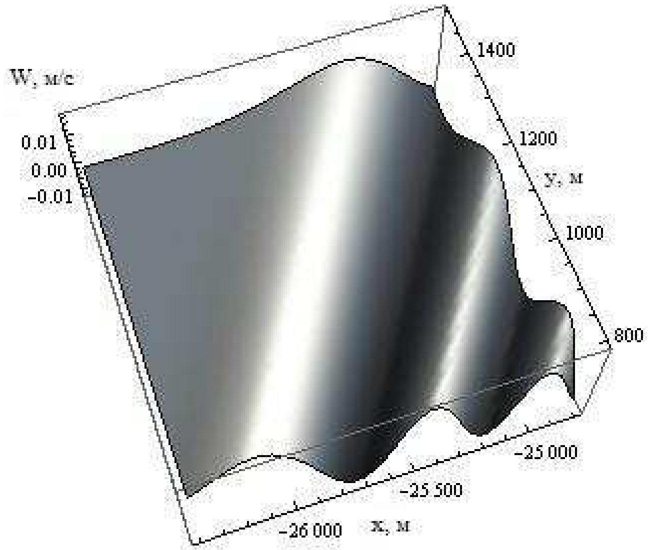

Fig. 4. Evolution of internal gravity wave packet in non-stationary stratified ocean.

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