## WAVE DYNAMICS OF STRATIFIED OCEAN WITH NON-UNIFORM DEPTH

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### **ABSTRACT**

The paper is devoted to the research of the processes of disturbance and propagation of the internal gravity waves (IGW) within the vertically stratified medium with non-uniform depth, to development of the asymptotic methods being by the generalization of the space-time ray-tracing method (the method of the geometrical optics, the modified WKBJ method). Analytical and numerical results of IGW calculations obtained with the use of asymptotic formulas for the real parameters of the ocean are presented.

### PROBLEM FORMULATION

There is growing interest in the mathematical modeling of the dynamics of internal gravity waves (IGW) in inhomogeneous stratified natural environments, due to the problems of geophysics, oceanography, atmospheric physics protection and study of the environment, the operation of complex hydraulic structures, including offshore oil facilities and a number of other actual problems of science and technology. This paper sets forth the fundamental problems of mathematical modeling of the dynamics of IGW in the ocean of variable depth and analytical results obtained compared with the simulation results and the measured data. IGW in the ocean have been studied for a long time, and on this subject published a significant number of jobs.

However, in recent years the interest in them to some degree fades that can be measured by the total number of publications devoted to this subject. However, now there are new directions in the study of IGW, which had not previously mentioned. Firstly, it became clear that in the field of internal waves may appear abnormally large short-killer waves, the nature of which resembles the nature of the killer waves on the sea surface. Second, the shear flow in the internal waves lead to large bending moments on the support of oil platforms that have led to the deformation of underwater technology designs in a number of areas of the oceans. Now, a system of intensive monitoring of IGW, similar detection system of tsunami waves. Third, IGW can cause transport of bottom sediments in the deep sea, where the effect of surface waves on the bottom is minimal. Finally, the classic problem of the effects of IGW on the sea and ocean surface are still actual.

# MAIN RESULTS AND DISCUSSIONS

IGW dynamics in the ocean are strongly influenced by the heterogeneity, the intermittence of hydro-physical fields and change the bottom topography. In this case, the exact analytical solutions of wave problems can only be obtained if the water density distribution and the shape of the bottom covered very simple model functions. When the characteristics of the environment and the boundaries are arbitrary, it is possible to build only the numerical solution of such problems.

However, numerical simulation does not allow qualitatively analyze the characteristics of wave fields, especially over long distances, it is necessary to solve, for example, the problem of detecting IGW remote methods, including by means of aerospace radar. In this case, a description and analysis of wave dynamics must be based on asymptotic models and analytical methods for solving them. Of particular interest to the modeling of the dynamics of IGW is associated with intensive development of the Arctic and its natural resources, as they have not been studied in the area of the oceans.

Universal character set out in the report of the asymptotic methods of modeling IGW can effectively calculate the wave field, and, in addition, qualitatively analyze the solutions obtained. This opens up opportunities analysis of the wave as a whole, which is important for the correct formulation of mathematical models of wave dynamics and the rapid assessments of in-situ measurements of wave fields in the marine environment.

The special role of the developed asymptotic methods is due to the fact that the parameters of natural stratified media, usually known approximately, and attempts to exact numerical solutions to the equations of hydrodynamics source using such parameters may lead to a significant loss of accuracy of the results. Figures show the results of the asymptotic (fig.1) and numerical (fig.2) modeling the dynamics of IGW over an uneven bottom, and the results of field observations of IGW in the ocean (fig.3).

The results show that the wave pattern with a strong rays structure can be observed in the real ocean in the study of the dynamics of IGW over an uneven bottom. In particular, analytical, numerical and field data show that the width of the wave beams decreases when approaching the shore. Fig. 2 shows the results of numerical simulation of the complete system of hydrodynamic equations describing the evolution of nonlinear wave disturbances over an uneven ocean floor (Bay of Biscay, summed over 60 IGW individual modes). As can be seen from presented results clearly identified rays structure of solutions. As the results show, the amplitude-phase structure of the wave fields is described quite well received in the asymptotic formulas. Fig.3 shows the results of tidal IGW field measurements in the same region of the World Ocean (Bay of Biscay).

These field data and numerical simulation results show that, indeed, the wave pattern obtained with a pronounced rays structure can be observed in a real ocean, especially in the study of the evolution of packets of internal gravity waves over an non-uniform bottom. In particular, analytical, numerical and IGW measurements data show that the width of the wave beams decreases when approaching the shore.

Formally, in the linear formulation, the width of the reflected beam of internal waves can be arbitrarily small for the corresponding relations between the parameters of the medium, such as stratification and the angle of the bottom. Near the coast there is a significant local increase in the amplitude of the waves. However, it is clear that in the real natural stratified environments (ocean, atmosphere), the energy of the wave field remains in these spatial domains of finite, because it includes non-linear mechanisms of dissipation and turbulent mixing of sea water.

The universal character of the asymptotic method proposed for modeling IGW far fields makes it possible to effectively calculate wave fields and, in addition, qualitatively analyze the obtained solutions. This method offers broad opportunities for the analysis of wave fields on a large scale, which is important for developing correct mathematical models of wave dynamics and for assessing in situ measurements of wave fields in the ocean. The particular role of the proposed asymptotic methods is determined by the fact that the parameters of natural stratified media are usually known approximately and attempts at their adequate numerical solution using the initial equations of hydrodynamics and such parameters may result in a notable loss of accuracy for the results obtained.

In addition to their fundamental significance, the obtained asymptotic models are also important for applied investigations, since the proposed method of geometrical optics allows solution of a wide spectrum of problems related to modeling wave fields. The obtained asymptotic solutions are uniform and allow far internal gravity wave fields to be described both near and far from turning and focal points, wave fronts and caustics. In such a situation, the description and analysis of wave dynamics may be realized through developing asymptotic models and using analytical methods for their solution based on the proposed WKBJ modified method.

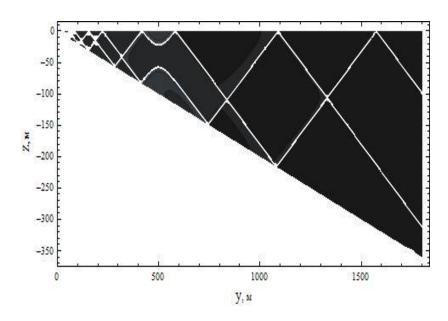


Fig. 1. Rays structure of IGW in stratified ocean with non-uniform depth: analytical results of IGW velocity amplitude calculation.

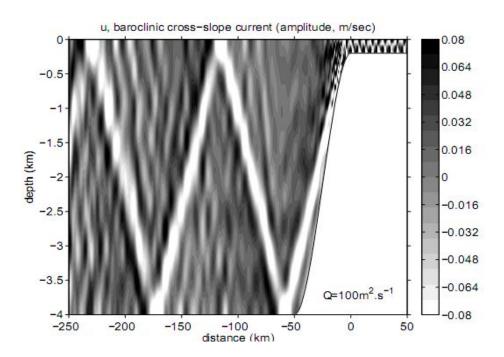


Fig. 2. Rays structure of IGW in stratified ocean with non-uniform depth (Bay of Biscay): numerical results of IGW velocity amplitude calculations.

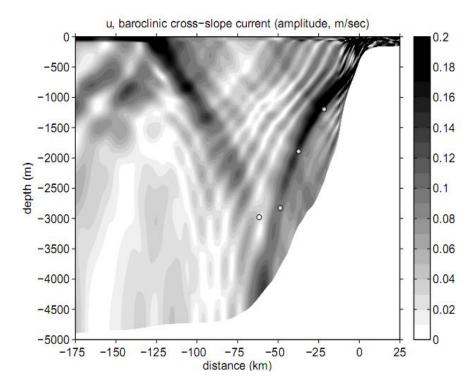


Fig. 3. Rays structure of IGW in stratified ocean with non-uniform depth (Bay of Biscay): measurements data of IGW velocity amplitude..

### **ACKNOWLEDGMENTS**

The results presented in the paper have been obtained by research performed under projects supported by the Russian Foundation for Basic Research (No. 14-01-00466), Program of the Russian Academy of Sciences "Fundamental Problems of Oceanology: Physics, Geology, Biology, Ecology".

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